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# Weakly nonlinear dispersion and stop-band effects for periodic structures

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## ABSTRACT

Continua and structures composed of periodically repeated elements (cells) are used in many fields of science and technology. Examples of continua are composite materials, consisting of alternating volumes of substances with different properties, mechanical filters and wave guides. Examples of engineering periodic structures include some building frames, bridge trusses, cranes, railway tracks, and compound pipes. Thus dynamic analysis of spatially periodic structures is relevant for many applications, and attracts much attention. An essential feature of periodic structures is the presence of frequency band-gaps, i.e. frequency ranges in which elastic waves cannot propagate. Most existing analytical methods in the field are based on Floquet theory [1]; e.g. this holds for the classical Hill's method of infinite determinants [1,2], and the method of space-harmonics [3]. However, application of these methods for studying nonlinear problems is impossible or cumbersome, since Floquet theory is applicable only for linear systems. Thus the nonlinear effects for periodic structures are not yet fully uncovered, while at the same time applications may demand effects of nonlinearity on structural response to be accounted for.

The paper deals with analytically predicting dynamic response for nonlinear elastic structures with a continuous periodic variation in structural properties. Specifically, for a Bernoulli-Euler beam with a spatially continuous modulation of structural properties in the axial direction, not necessarily small, we consider the effects of weak nonlinearity on the dispersion relation and frequency band-gaps. A novel approach, the *Method of Varying Amplitudes* [4], is employed. This approach is inspired by the method of direct separation of motions [5], and may be considered a natural continuation of the classical methods of harmonic balance [2] and averaging [6]. It implies representing a solution in the form of a harmonic series with varying amplitudes, but, in contrast to averaging methods, the amplitudes are not required to vary slowly. The approach is strongly related also to Hill's method of infinite determinants [1,2], and to the method of space-harmonics [3]. As a result, a shift of band-gaps to a higher frequency range is revealed, while the width of the band-gaps appears relatively insensitive to (weak) nonlinearity. The results are validated by numerical simulation, and explanations of the effects suggested. The work is carried out with financial support from the Danish Council for Independent Research and COFUND: DFF – 1337-00026.

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